Guide for electrical design engineers

Leonardo ENERGY

Power Quality

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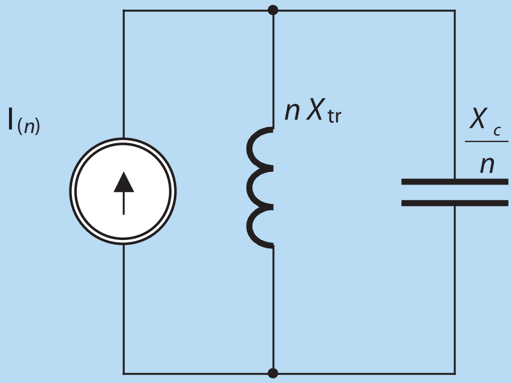
Centralised reactive power compensation in case of harmonic pollution



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Power Quality

Problem

A 400V switchboard is fed from the transformer with rated power S_N=400kVA and short-circuit voltage $e_{\%}$=4.5%. The switchboard supplies loads with total active power ranging from P=190 kW to 208kW and a power factor varying from 0.62 to 0.68. The load current is distorted; maximum values of harmonic currents are given in the table below.

n	5	7	11	13
$I_{(n)}$ [A]	10	7.14	5.13	4.1

Determine the maximum and minimum compensator power in order to compensate reactive power to the power factor value of 0.92. Check the voltage distortion is maintained within allowed limits and calculate the series reactor impedance, if needed.

For the purpose of this calculation the power system short-circuit capacity, resistances of components and compensator active power losses can be neglected. Assume the supply voltage is non-distorted.

Reactive power compensation

The reactive power needed to compensate a load can be calculated from the reactive power balance. As the active power does not change in result of compensation, the following formula can be used

$$Q_k = P\left(\tan\varphi_n - \tan\varphi_d\right)$$

where: Q_k – reactive power necessary for compensation; P – the load active power; $\tan\varphi_n$ – non-compensated tangent value; $\tan\varphi_d$ – tangent value that should be achieved by the compensation.

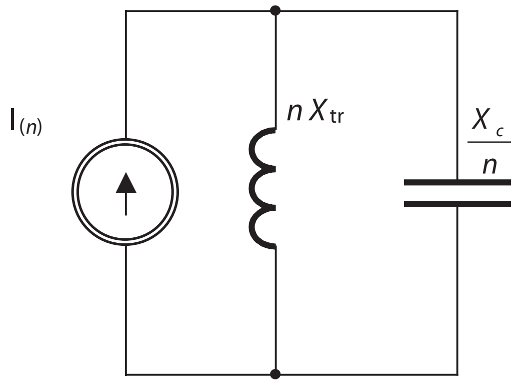
As the active power varies also the power factor and the load reactive power are variable. Since there is no information on the correlation between the power factor and the active power, we shall assume extreme cases and calculate both the maximum and minimum compensating power for the given tangent value. The maximum power is drawn at the maximum active power and the maximum difference between tangent values, i.e. at the minimum power factor of 0.62. Correspondingly, the minimum power occurs for the minimum active power and maximum power factor 0.68. Therefore, we obtain

$$Q_{k,\min} = 190\left(1.078 - 0.426\right) = 123.93 \quad \text{kVAr}$$
$$Q_{k,\max} = 208\left(1.265 - 0.426\right) = 174.61 \quad \text{kVAr}$$

Thus the compensator reactive power has a certain constant value of Q_{st} = 123,93 kVAr, whereas the regulation range is Q_{reg} = 50,61 kVAr.

Since the load current is distorted, there is a possibility a resonance may occur. Moreover, neglecting the resistance implies that attenuation introduced by certain elements will not be taken into account. It should be noted that harmonic currents values are small compared to the 400 kVA transformer rated current.

For high harmonic frequencies the transformer, capacitor bank and the load form a parallel circuit as shown in figure below.



Where: $I_{(n)}$ – n-th harmonic source current; X_{tr} – the transformer reactance calculated for the first harmonic; X_c – the compensator reactance the first harmonic; n – relative frequency (with respect to the fundamental — a harmonic order).

In this case the equivalent impedance (actually the reactance, assuming the resistance is neglected) is

$$Z_z(n) = \frac{-nX_c X_{tr}}{n^2 X_{tr} - X_c}$$

The resonance conditions are related to the zeros of the numerator or denominator. For positive values of circuit components only the numerator can become zero. This occurs for

$$n^2 X_{tr} = X_c$$

and is the condition of a parallel resonance; the resonance frequency is

$$n_r = \sqrt{\frac{X_c}{X_{tr}}}$$

The reactance values can be determined from simple formulas:

$$X_{c,\min} = \frac{U^2}{Q_{k,\max}} = \frac{400}{147.61 \cdot 10^3} = 0.916 \quad \Omega$$

$$X_{c,\max} = \frac{U^2}{Q_{k,\max}} = \frac{400}{123.93 \cdot 10^3} = 1.291 \quad \Omega$$

$$X_{tr} = \frac{e_{\%}}{100}\frac{U^2}{S_{tr}} = 0.045 \cdot \frac{400}{400 \cdot 10^3} = 18 \quad \text{m}\Omega$$

As the compensator reactive power varies also its reactance does and, consequently, the resonance frequency is varying. Substituting the above values we determine the maximum and minimum resonance frequency:

- $n_{r,\,max} = 8.47$ for $Q_{r,\,min}$, $X_{r,\,max}$
- $n_{r,\,min} = 7.13$ for $Q_{r,\,max}$, $X_{r,\,min}$

Since the 7th harmonic is present in the load current, a resonance can occur when the capacitor bank is loaded with maximum power. This will result in an increase in the 7th harmonic current and, consequently, significant voltage distortion at the point of common coupling (at the switchboard bus-bars).

In order to check whether the capacitor bank operation is possible under such conditions, the maximum total harmonic voltage distortion THD should be calculated. The resonance will obviously occur at the frequency n_r equal 7.13, i.e. for the capacitor maximum reactive power. Knowing maximum harmonic current values, the voltage drops and hence the voltage THD can be calculated. The table below shows the values calculated for each harmonic.

n	5	7	11	13
$I_{(n)}$ [A]	10	7.14	5.13	4.1
$Z_{(n)}$ [Ω]	0.177	3.365	0.144	0.1
$U_{(n)}$ [V]	1.768	24.03	0.738	0.414

Total harmonic voltage distortion THD can be determined from these values according to the definition

$$\text{THD} = \frac{\sqrt{\sum U_{(n)}^2}}{U_{N(1)}} \cdot 100\%$$

where $U_{N(1)}$ – rms value of the phase voltage (the first harmonic); $U_{(n)}$ – rms value of the n-th harmonic voltage drop.

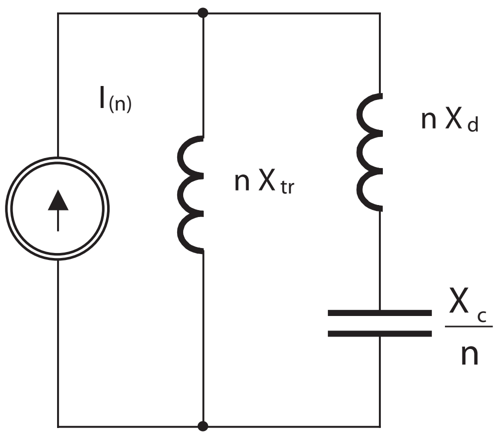
The calculated value of total harmonic distortion THD will be

$$\text{THD} = \frac{\sqrt{1.768^2 + 24.03^2 + 0.708^2 + 0.414^2}}{230} \cdot 100\% = 10.48\%$$

This value exceeds the limit of 8% set forth in standard EN 50160 with regard to low voltage. Such high a voltage distortion occurs only at the maximum capacitor bank power, therefore either the power needs to be reduced or a series reactor should be connected.

Voltage harmonics suppression

Connecting a series reactor creates conditions for series resonance. The equivalent circuit for this case is shown in figure below.



Where X_d – is the reactance of the series reactor.

The equivalent impedance of this circuit is

$$Z_z(n) = \frac{nX_{tr}\left(n^2 X_d - X_c\right)}{n^2\left(X_{tr} + X_d\right) - X_c}$$

Two options are possible when the numerator or denominator equals zero.

The nominator becomes zero if the relationship

$$n^2 X_d - X_c = 0$$

is satisfied, which is the condition of series resonance. If the resonance frequency is known the reactor reactance can be calculated from

$$X_d = \frac{X_c}{n_s^2}$$

For the denominator we obtain the relationship

$$n^2\left(X_{tr} + X_d\right) - X_c = 0$$

which is the condition of parallel resonance. The resonance frequency is

$$n_r = \sqrt{\frac{X_c}{X_{tr} + X_d}}$$

It should be noted that the parallel resonance frequency has been shifted toward lower frequencies compared to that of the circuit without the series reactor. Therefore, when choosing the reactor the series resonance frequency should be selected to match the lowest harmonic present — in this case the 5th.

Choosing the series resonance frequency for e.g. the 7th harmonic will shift the parallel resonance frequency towards the 5th harmonic.

This may increase the voltage distortion compared to that of the circuit without the reactor. As an example, choosing the reactor for the 7th harmonic and using the values given in this problem, we obtain

$$X_d = \frac{X_c}{n_s^2} = \frac{0.919}{7^2} = 18.55 \quad \text{m}\Omega$$

But the parallel resonance will occur for the frequency

$$n_r = \sqrt{\frac{X_c}{X_{tr} + X_d}} = 4.99$$

and, in consequence, the voltage distortion THD will be approximately 146%.

Therefore the reactor shall be chosen for the frequency equal to the 5th harmonic. Then we obtain

$$X_d = \frac{X_c}{n_s^2} = \frac{0.919}{5^2} = 36.76\ \text{m}\Omega$$

The parallel resonance frequency will vary within the range

$$n_{r,\text{max}} = 4.86 \quad n_{r,\text{min}} = 4.09$$

It is thus sufficiently far from the current harmonics (namely the 5th harmonic).

Now we calculate the maximum voltage distortion THD which will occur for the minimum compensator power, i.e. at the resonant frequency 4.86. The calculations are tabulated below:

n	5	7	11	13
$I_{(n)}$	10	7.14	5.13	4.1
$Z_{(n)}$	0.448	0.046	0.117	0.144
$U_{(n)}$	4.48	0.327	0.6	0.592

Total voltage distortion THD is

$$\text{THD} = \frac{\sqrt{4.48^2 + 0.327^2 + 0.6^2 + 0.592^2}}{230} \cdot 100\% = 1.99\%$$

which is far below the admissible value.

The graph of the modulus of impedance versus frequency is shown in figure 1b.

The correct selection of the compensator components, i.e. the capacitors and reactor, requires determining their operating conditions.

The current in the compensator branch can be found employing Kirchhoff current law.

$$I_{f(n)} = I_{(n)} \frac{n^2 X_{tr}}{n^2 \left(X_{tr} + X_d \right) - X_c}$$

where: $I_{(n)}$ – rms n-th harmonic current, X_d, X_{tr}, X_c – components reactances for the fundamental harmonic.

The fundamental harmonic current we calculate from the formula:

$$I_{f(1)} = \frac{U}{\sqrt{3}\left(X_c - X_d \right)} = 262.53\ A$$

The worst-case calculation results are tabulated below

n	5	7	11	13
$I_{(n)}$	10	7.14	5.13	4.1
$I_{f(n)}$	59.81	4.54	2.1	1.57

The rms current value we calculate from the formula:

$$RMS = \sqrt{\sum I_{(n)}^2}$$

This yields:

$$RMS = \sqrt{262.53^2 + 59.81^2 + 4.54^2 + 2.1^2 + 1.57^2} = 269.31 \quad \text{A}$$

This value should be smaller than the admissible current value

$$I_{rms} = 1.8 \cdot I_{CN}$$

where I_{CN} is the rated capacitor current.

The capacitor voltage increase can be found from the Kirchhof voltage law, or using the calculated values of the compensator branch current. Thus we obtain:

n	5	7	11	13
$I_{f(n)}$	59.81	4.54	2.1	1.57
$X_{C(n)}$	0.183	0.131	0.083	0.07
$U_{C(n)}$	10.94	0.59	0.17	0.11

From the voltage divider the capacitor fundamental harmonic voltage rms value is

$$U_{C(1)} = \frac{X_c}{X_c - X_d} U = 416.67\,V$$

which yields the rms value:

$$RMS = \sqrt{240.56^2 + 10.94^2 + 0.59^2 + 0.17^2 + 0.11^2} = 240.81 \quad \text{V}$$

This value should be less than the maximum permissible continuous voltage, determined as

$$U_{C,rms} < 1.2 U_{CN}$$

where U_{CN} – the rated capacitor voltage.

The peak capacitor voltage is determined from the formula

$$MAX = \sum U_{(n)}$$

This yields

$$\text{MAX} = 240.56 + 10.94 + 0.59 + 0.17 + 0.11 = 252.37 \quad \text{V}$$

This value should be less than the maximum permissible peak voltage, which is

$$U_{C,\max} < 1.1 U_{CN}$$

Using these parameters we can specify the size and type of capacitors and reactors appropriate for the given compensator requirements.